Class:	EE 420L – Engineering Electronics II		Semester:	Spring 2023
Points		Document author:	Abraham Castaneda Dylan Cazares Daniela Nikoloska Maxwell Stoneham	
		Author's email:	Castaa7@unlv.nevada.edu cazard1@unlv.nevada.edu nikolosk@unlv.nevada.edu stonham@unlv.nevada.edu	
		Document topic:	Postlab 10	
Instructor's comments:				

1. Introduction / Theory of Operation

Butterworth filters, developed by the British engineer and physicist Stephen Butterworth, is a signal processing filter that is designed to have a frequency response that is as flat as possible in the passband. A Butterworth filter is a type of lowpass filter described by the following equation

$$|H(\omega)| = \frac{1}{\sqrt{\left(1+\left(\frac{\omega}{\omega c}\right)^{2N}\right)}}$$

Equation 1: General form transfer function H(ω)

Where N is called the "order" or number of poles of the filter, ω_c is the filters -3dB cutoff frequency and ω is angular frequency. For this project, we are implementing a third order lowpass Butterworth filter that is described by the following transfer function. Which we defined

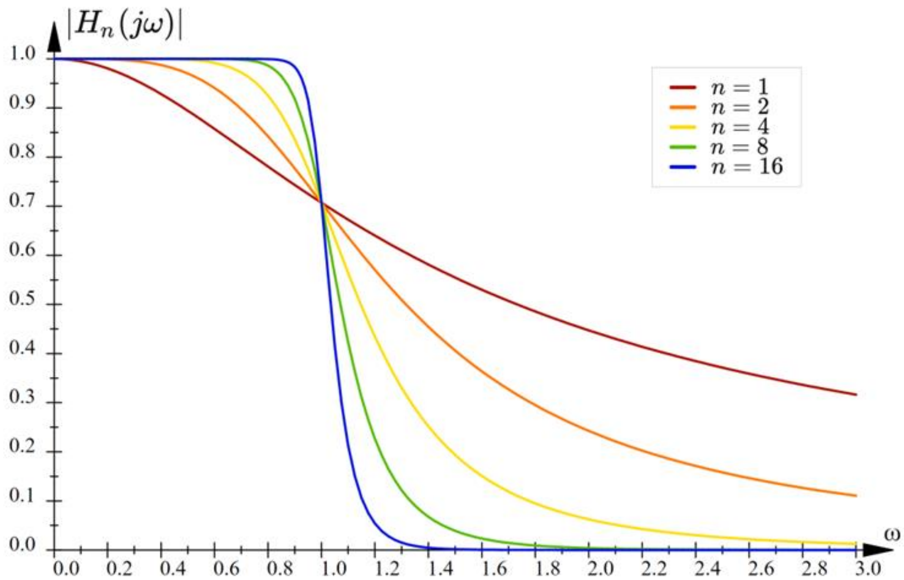
$$H(s) = \frac{a0}{s^3 + a2s^2 + a1s + a0}$$

the coefficients as

$$a_2 = 1.257 \times 10^6$$
$$a_1 = 789.6 \times 10^{10}$$
$$a_0 = 248.1 \times 10^{15}$$

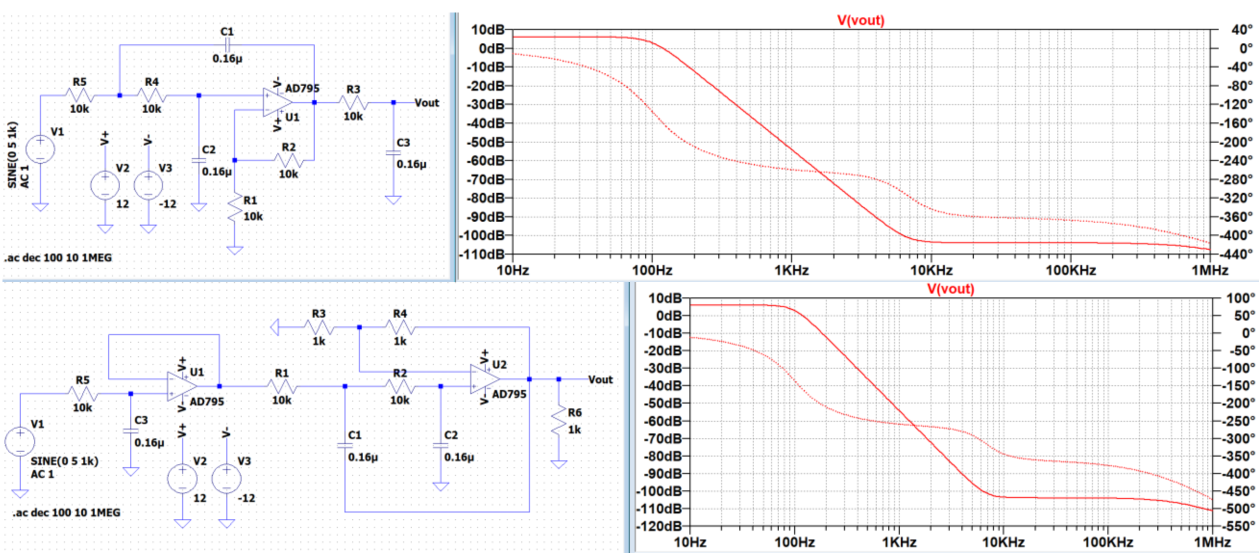
We will be using block diagrams to derive the transfer function which will determine how the circuit is created. This project will be using the LM741 operational amplifier, 1nF capacitors, and our resistor values will be determined by our calculations gathered from the block diagram derivation.

The focus on designing a Butterworth filter is to have a flat frequency response in the passband, and a smooth roll-off after it reaches the cutoff frequency is reached, as shown below. We notice that as the nth order Butterworth filter increases, the flatter the response can be at a given gain, and the steeper the roll off will be after the cutoff.



Below are two examples of a general third order Butterworth filter set at a cutoff frequency of 100kHz. Using resistor values of 10k, with a cutoff frequency of 100kHz, we can solve for our capacitor values using

$$fc = \frac{1}{2\pi RC}\ Hz$$



However, our focus for this project would be the design of a Butterworth filter using 1nF capacitors by casoding integrators in series to achieve this same frequency response.

2. Description of Experiments

2.1 Transposed Direct Form II Block Diagram

Designing the Butterworth filter, we needed to create a Transposed Direct Form II Block diagram. To create the TDFII block diagram for H(s) we need to derive it into its input and output stages by. We know that the transfer function H(s) is the quotient of X(s) and Y(s), where to get the output of the transfer function we to have the output y(t) = x(t)h(t).

$$H(s) = \frac{X(s)}{Y(s)} = \frac{a0}{s^3 + a2s^2 + a1s + a0}$$

$$Y(s) + \frac{Y(s)a2}{s} + \frac{Y(s)a1}{s^2} + \frac{Y(s)a0}{s^3} = \frac{X(s)a0}{s^3}$$

$$y(t) + a2\int y(t) + a1\int^2 y(t) + a0\int^3 y(t) = a0\int^3 x(t)$$

$$y(t) = a0\int^3 x(t) - a2\int y(t) - a1\int^2 y(t) - a0\int^3 y(t)$$

Equation X – Derivation of output y(t)

Now with this equation we can create the TDFII block diagram using integrators.

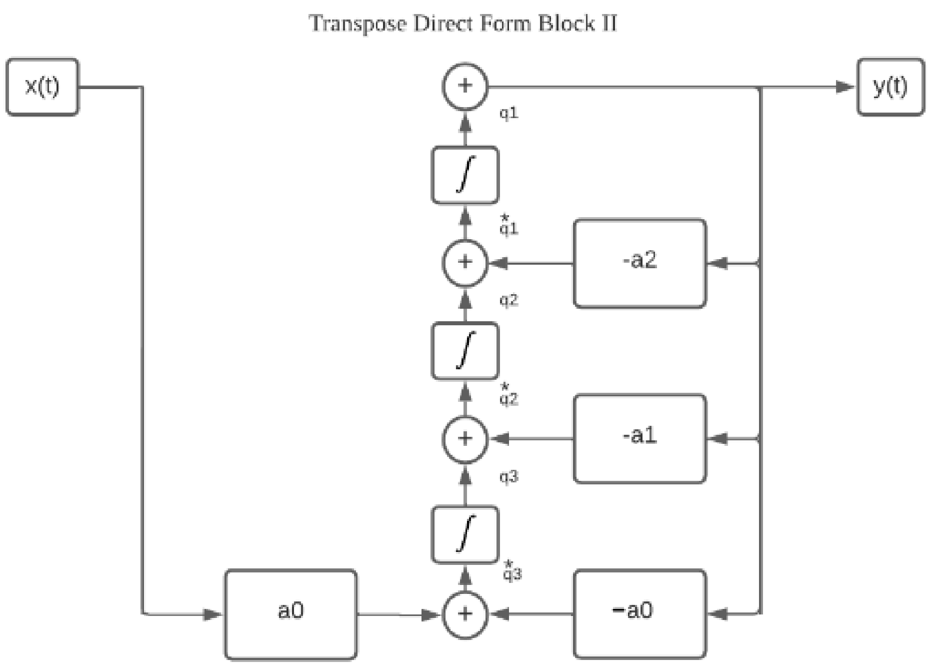


Figure 1 – Transposed DFII of Transfer Function H(s)

```
a0 = 248.1*10^15;
a1 = 789.6*10^9;
a2 = 1.257*10^6;

num = a0;
den = [1 a2 a1 a0];
H = tf(num,den)

bode(sys)
grid
```

```
>> BWF

H =

                    2.481e17
  ------------------------------------------
  s^3 + 1.257e06 s^2 + 7.896e11 s + 2.481e17

Continuous-time transfer function.
```

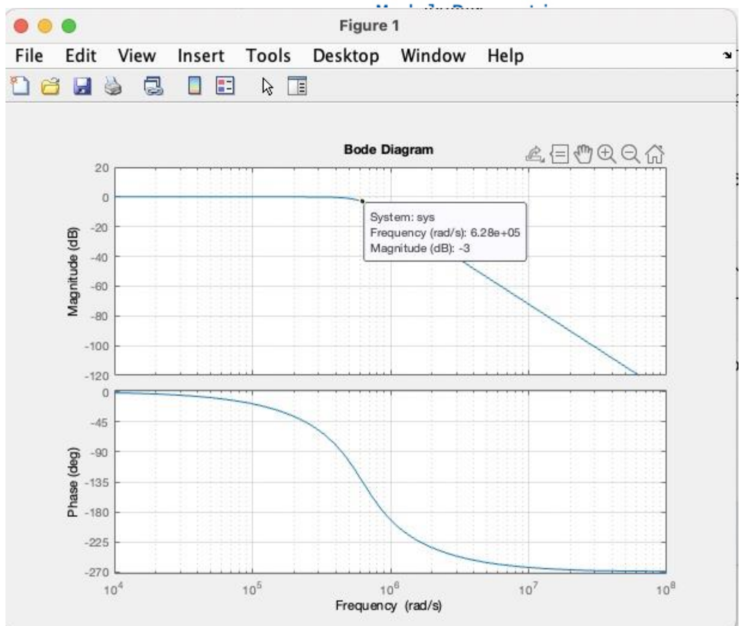


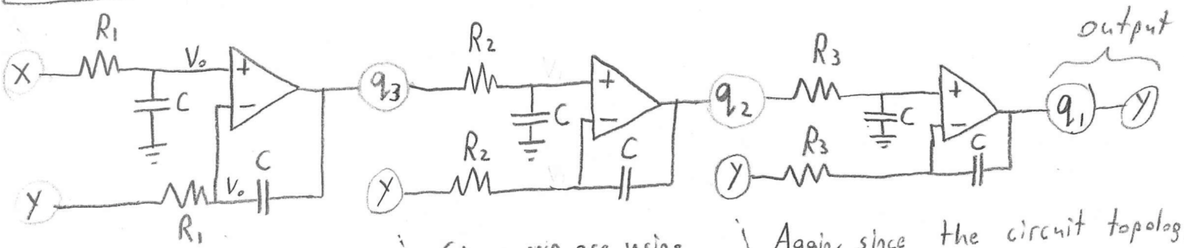
Figure 2 – MATLAB Plot of H(s)

2.2 RC Integrators Hand Calculations

To find the resistor values, assuming capacitors values of 1nF, the gains of the three sections are cascaded and solved for (see next page):

We want $y = a_0\iiint x - a_0\iiint y - a_1\iint y - a_2\int y$ where $a_2 = 1.257\times10^{6}$, $a_1 = 789.6\times10^{9}$, $a_0 = 248.1\times10^{15}$

* All C's are 1nF *



$$V_o = x\left(\frac{1/sC}{R_1 + 1/sC}\right) = x\left(\frac{1}{sR_1C+1}\right)$$

$$\frac{y - V_o}{R_1} = \frac{V_o - q_1}{(1/sC)}$$

$$q_3 = V_o - \left(\frac{y - V_o}{sR_1C}\right) = V_o + \frac{V_o}{sR_1C} - \frac{y}{sR_1C}$$

$$= V_o\underbrace{\left(1 + \frac{1}{sR_1C}\right)}_{\frac{sR_1C+1}{sR_1C}} - \frac{y}{sR_1C}$$

$$= x\left(\frac{1}{sR_1C+1}\right)\left(\frac{sR_1C+1}{sR_1C}\right) - \frac{y}{sR_1C}$$

$$q_3 = \frac{x}{sR_1C} - \frac{y}{sR_1C} = \frac{1}{R_1C}\left(\int x - \int y\right)$$

Since we are using the same circuit in this section, we know that q_2 will be:

$$q_2 = \frac{1}{R_2C}\left(\int q_3 - \int y\right)$$

Again, since the circuit topology is the same, we know:

$$q_1 = \frac{1}{R_3C}\left(\int q_2 - \int y\right) = y$$

Expanding all terms:

$$y = \frac{1}{R_3C}\left(\int q_2 - \int y\right) = \frac{1}{R_3C}\left[\int \frac{1}{R_2C}\left(\int q_3 - \int y\right) - \int y\right]$$

$$= \left(\frac{1}{R_3C}\right)\left(\frac{1}{R_2C}\right)\iint q_3 - \left(\frac{1}{R_3C}\right)\left(\frac{1}{R_2C}\right)\iint y - \left(\frac{1}{R_3C}\right)\int y$$

$$\left(\frac{1}{R_3C}\right)\left(\frac{1}{R_2C}\right)\iint \frac{1}{R_1C}\left(\int x - \int y\right) = \left(\frac{1}{R_3C}\right)\left(\frac{1}{R_2C}\right)\left(\frac{1}{R_1C}\right)\iiint x - \left(\frac{1}{R_3C}\right)\left(\frac{1}{R_2C}\right)\left(\frac{1}{R_1C}\right)\iiint y$$

$$y = \underbrace{\left(\frac{1}{R_3C}\right)\left(\frac{1}{R_2C}\right)\left(\frac{1}{R_1C}\right)}_{a_0}\iiint x - \underbrace{\left(\frac{1}{R_3C}\right)\left(\frac{1}{R_2C}\right)\left(\frac{1}{R_1C}\right)}_{a_0}\iiint y - \underbrace{\left(\frac{1}{R_3C}\right)\left(\frac{1}{R_2C}\right)}_{a_1}\iint y - \underbrace{\left(\frac{1}{R_3C}\right)}_{a_2}\int y$$

$a_2 = \frac{1}{R_3C} = 1.257\times10^{6}$ $\quad a_1 = (1.257\times10^{6})\left(\frac{1}{R_2C}\right) = 789.6\times10^{9}$ $\quad a_0 = (789.6\times10^{9})\left(\frac{1}{R_1C}\right) = 248.1\times10^{15}$

$$R_3 = \frac{1}{(1.257\times10^{6})(10^{-9})} \approx 795.5$$

$$R_2 = \frac{1.257\times10^{6}}{(789.6\times10^{9})(10^{-9})} \approx 1591.9$$

$$R_1 = \frac{789.6\times10^{9}}{(248.1\times10^{15})(10^{-9})} \approx 3182.6$$

2.3 LTSpice Circuit Diagram

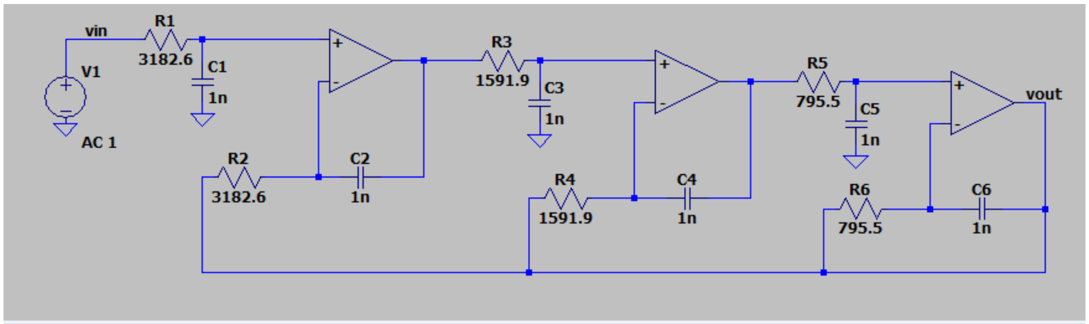


Figure 3 – LTSPICE circuit diagram

2.3.1 Using Ideal Components

Now that we know the general design of a 3rd order Butterworth filter using the affirmation block diagrams, we can create a circuit schematic with the resistance values that were calculated. Creating the circuit schematic, we used ideal components to get a "perfect world" output and frequency response to match our results from MATLAB. Doing AC Analysis on the circuit, we see that the frequency response matches perfectly to what a Butterworth filter does. With this, we can determine that the cutoff frequency for the bandpass is located at 100kHz which is where -3dB is located. After the cutoff frequency, the magnitude response starts to drop drastically as the frequency increases.

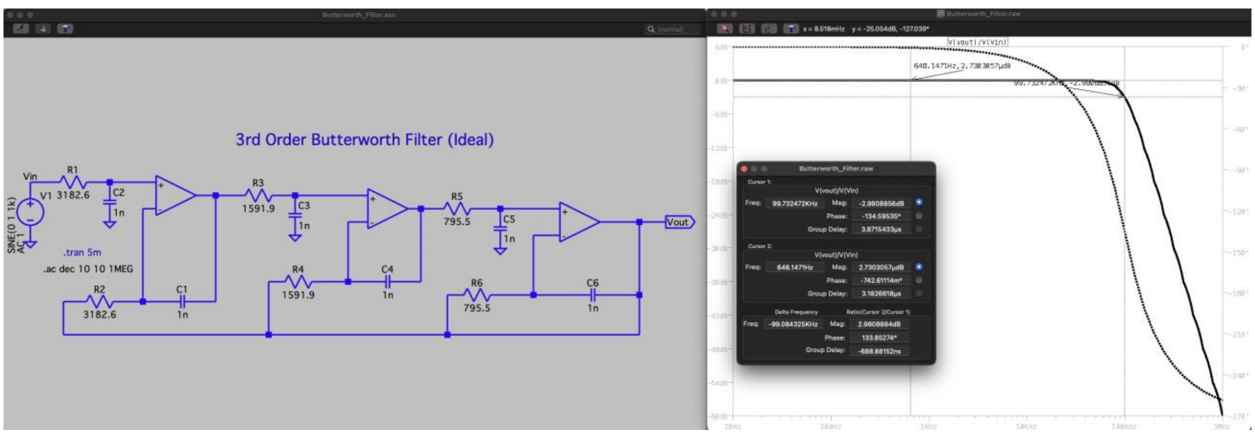


Figure 4 – Frequency Response of Ideal Butterworth Filter

Using a small input voltage of $20mV_{pp}$ at 1kHz we can see that the output voltage is nearly identical to the input voltage which we can see that there is a unity gain of 1. From our knowledge of Butterworth filters, this should hold true since it has a maximally flat amplitude until our desired cutoff frequency of 100kHz.

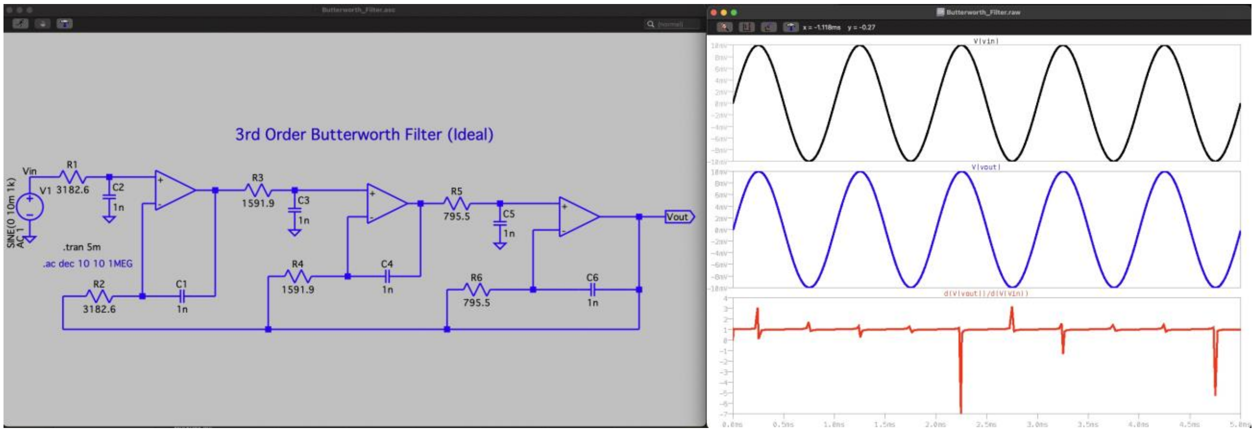


Figure 5 - Butterworth Schematic and Simulations at 1kHz

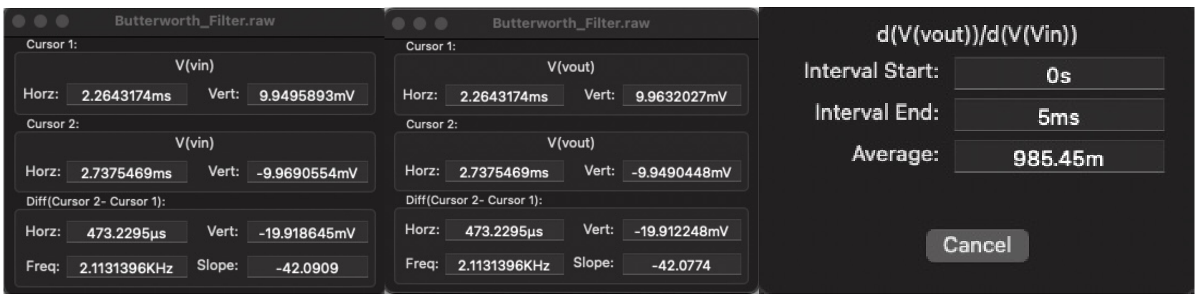


Figure 6 – Peak-to-Peak Voltages and Voltage Gain

Changing the frequency to 100kHz, we can see that the output voltage starting to decrease. From the 20mV_{pp} in the passband frequency to 14mV_{pp} in the cutoff frequency. The -3dB point is the frequency at which the magnitude of the filter's response is attenuated by 3dB (or about 30%). Above the cutoff frequency, the filter's response continues to decrease with increasing frequency, which is due to the filter's design characteristics. Looking at the voltage gain, we see that we have a gain of A_v = -0.223.

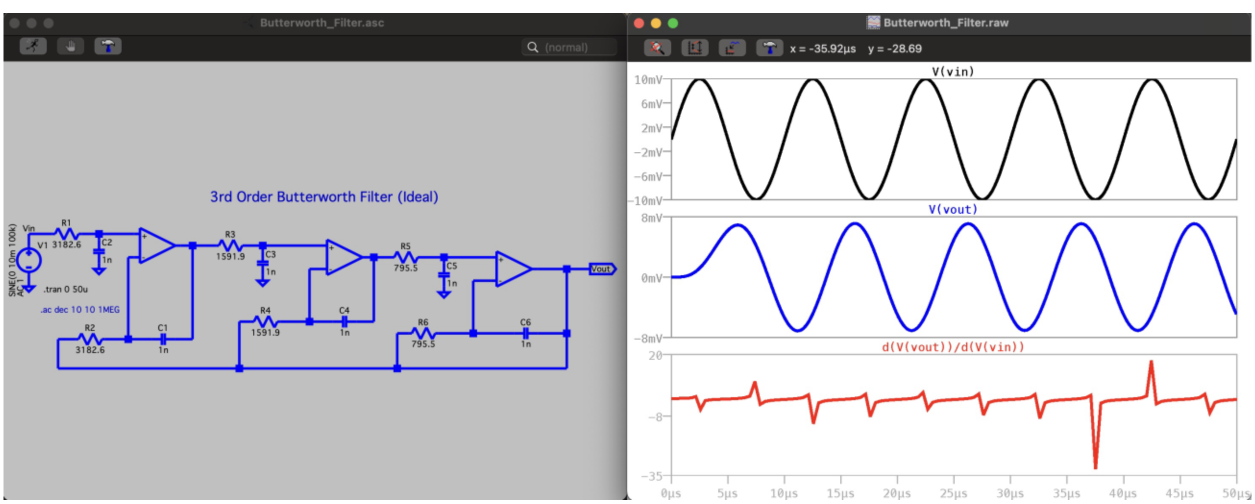


Figure 7 – Circuit Simulation at 100kHz

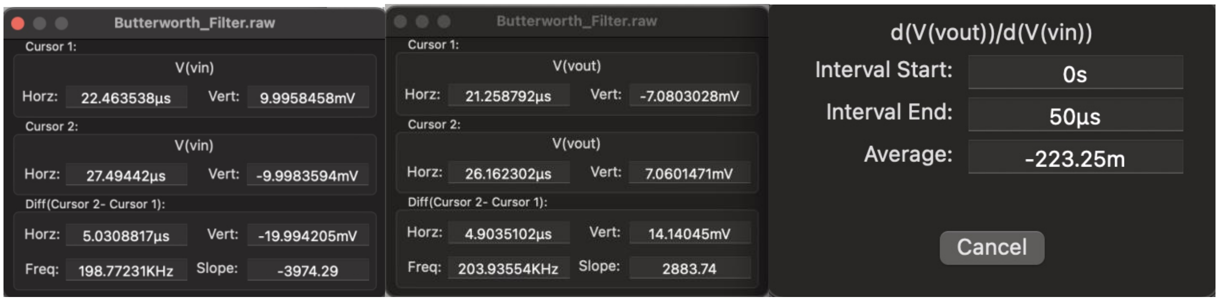


Figure 8 – Peak-to-Peak Voltages and Voltage Gain

Changing the input frequency to 1MHz, we can see that the output voltage has drasically dropped to 19.7µV_{pp} which our gain is now A_v = 156.7µV_{pp}.

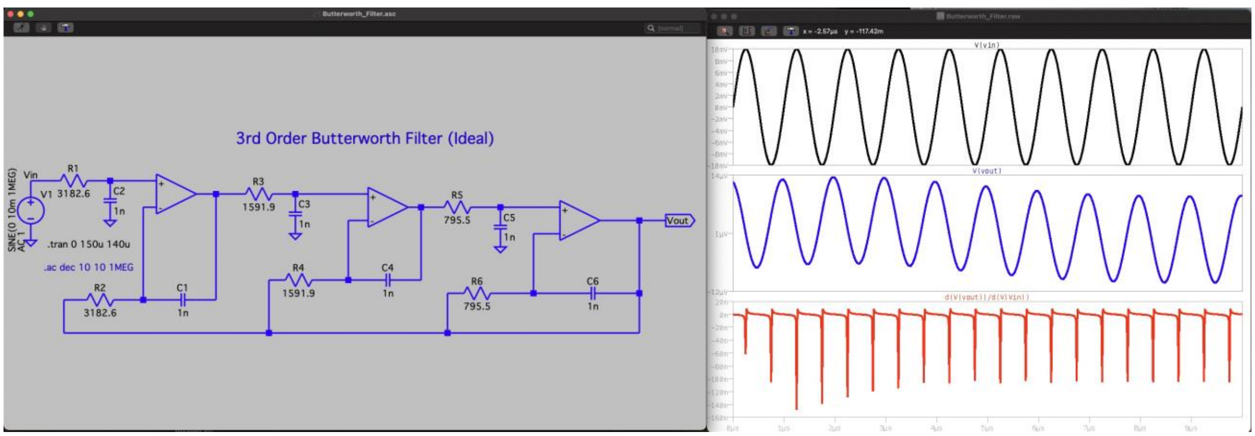


Figure 9 – Circuit Simulation at 1MHz

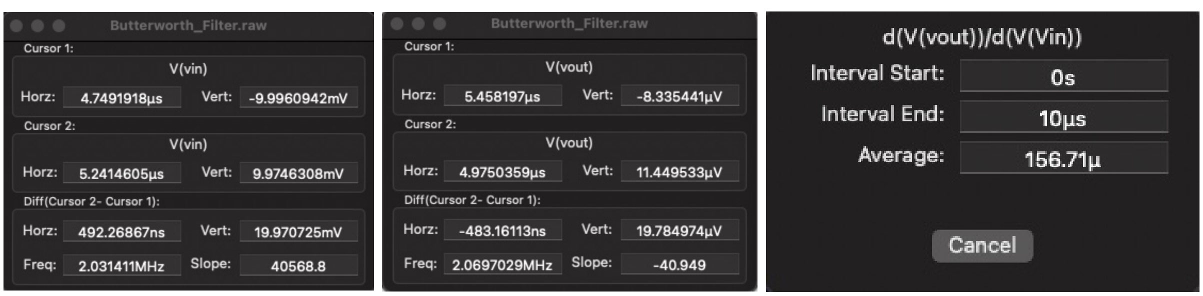


Figure 10 – Peak-to-Peak Voltages and Voltage Gain

2.3.2 Using Commercial Off the Shelf Components

Designing the filter using commercial off the shelf components, we see that the frequency response is nearly identical to that of the ideal Butterworth filter. However, there is a slight overshoot in the passband. This overshoot typically occurs at frequencies close to the cutoff frequency and is generally limited to a few percent of the maximum gain in the passband. After the overshoot, the magnitude of the frequency response starts to drop as intended, according to the order of the filter. This drop in magnitude is due to the roll-off of the filter, which is necessary to achieve the desired frequency response characteristics.

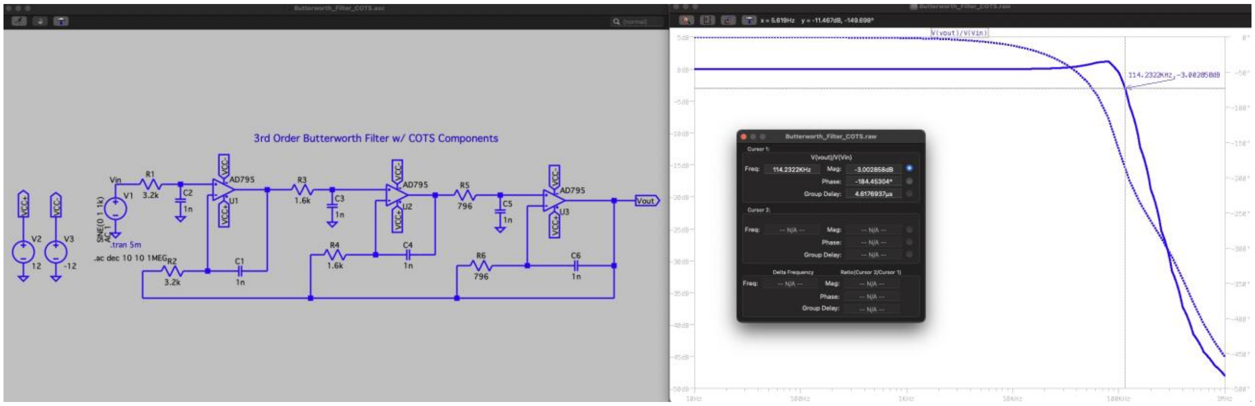


Figure 11 – Frequency Response of COTS components Butterworth Filter

Similar to the ideal filter, using a $20mV_{pp}$ input at 1kHz, we see that the output voltage has the same peak-to-peak voltage as the input. This same voltage should hold true based on the characteristics of a 3rd order Butterworth filtering up cut-off frequency.

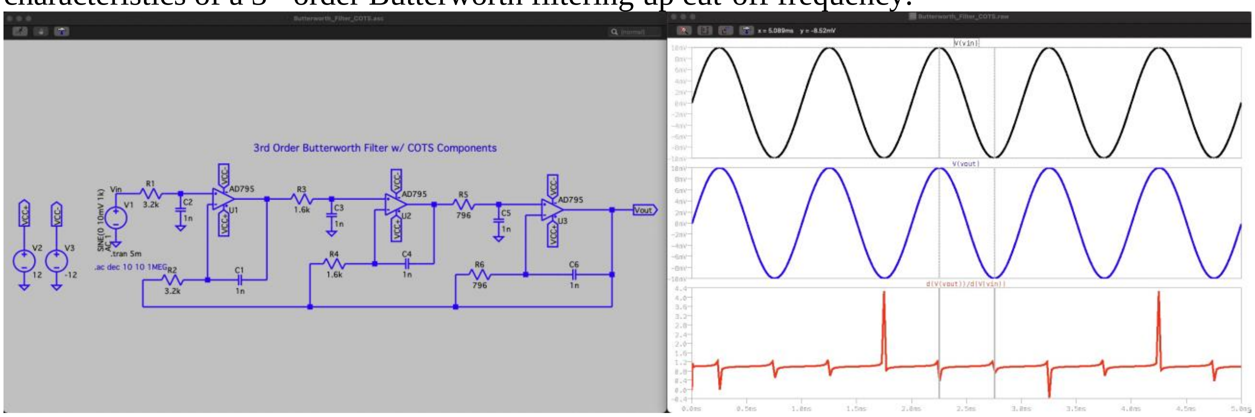


Figure 12 – Butterworth Schematic and Simulations

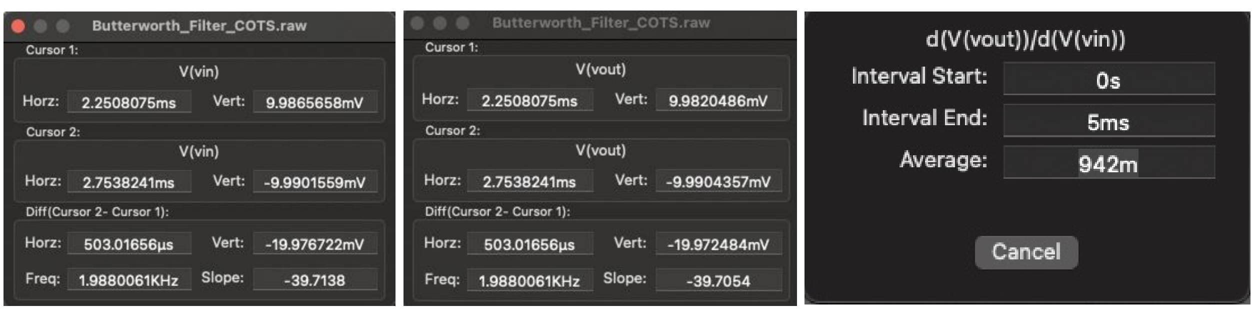


Figure 13 – Peak-to-Peak Voltages and Voltage Gain

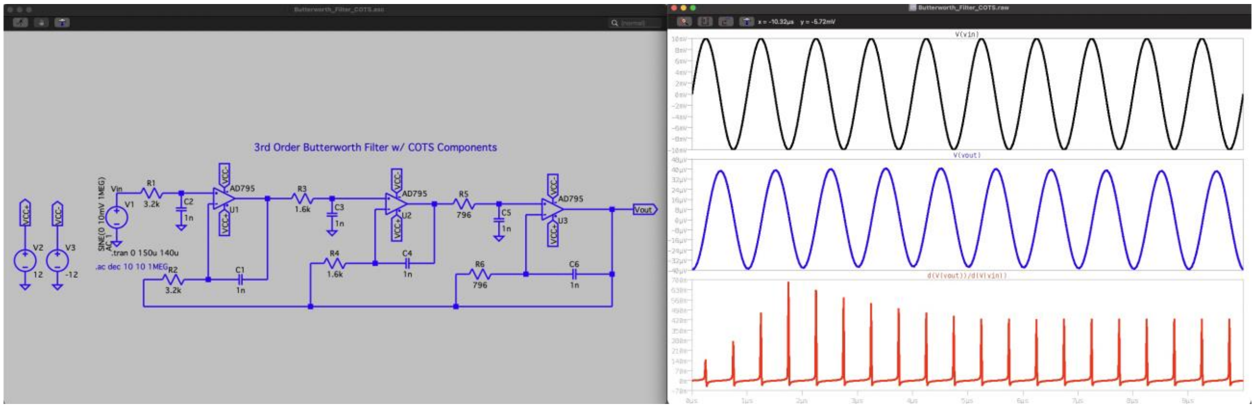


Figure 14 – Circuit Simulation at 1MHz

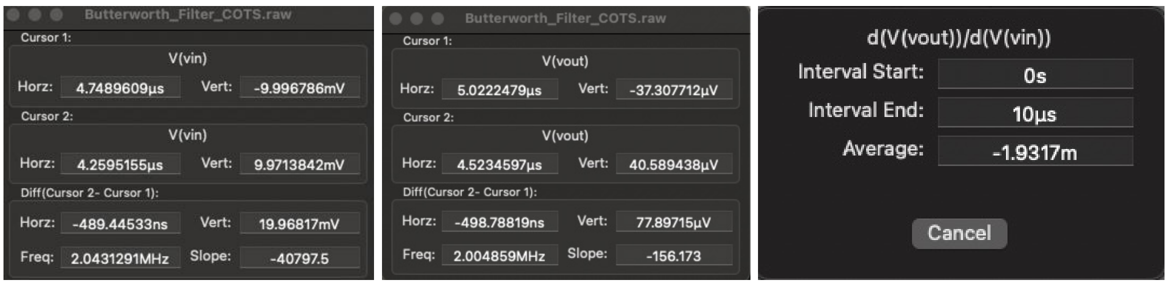
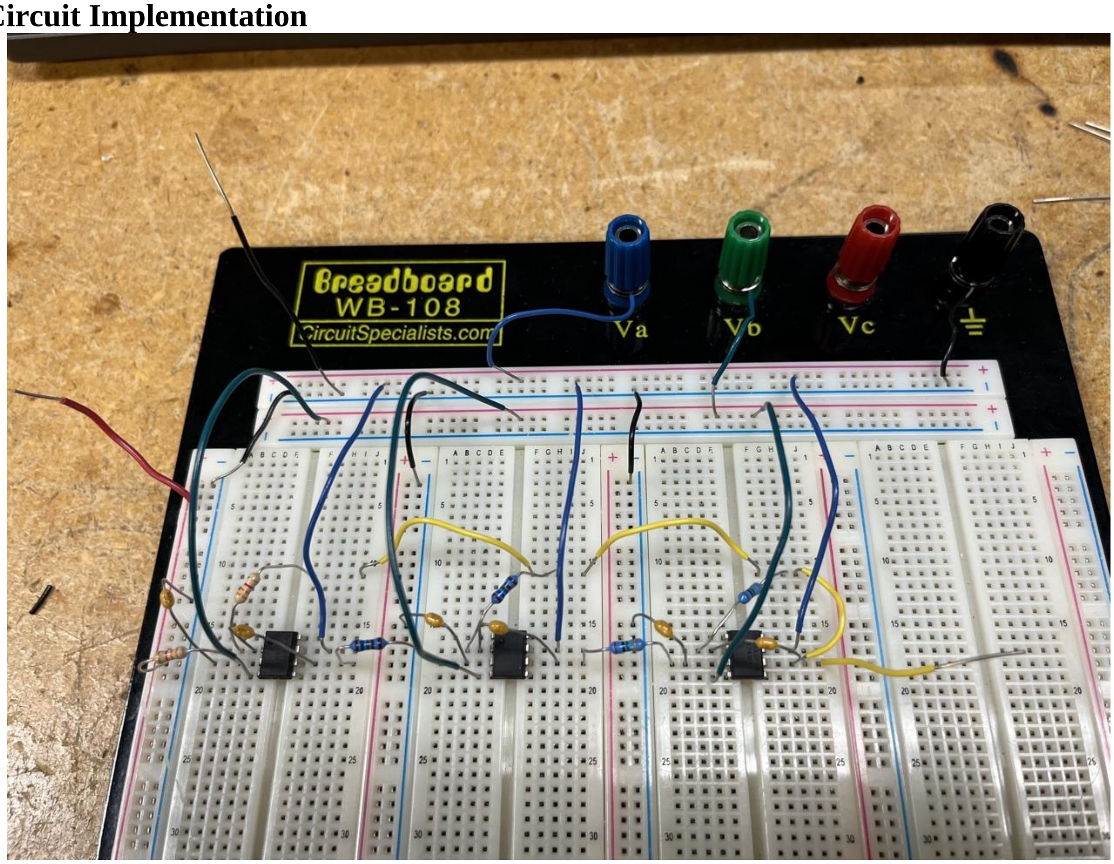


Figure 15 – Peak-to-Peak Voltages and Voltage Gain at 1MHz

2.4 Circuit Implementation



2.4.1 Frequency Response of Butterworth Filter

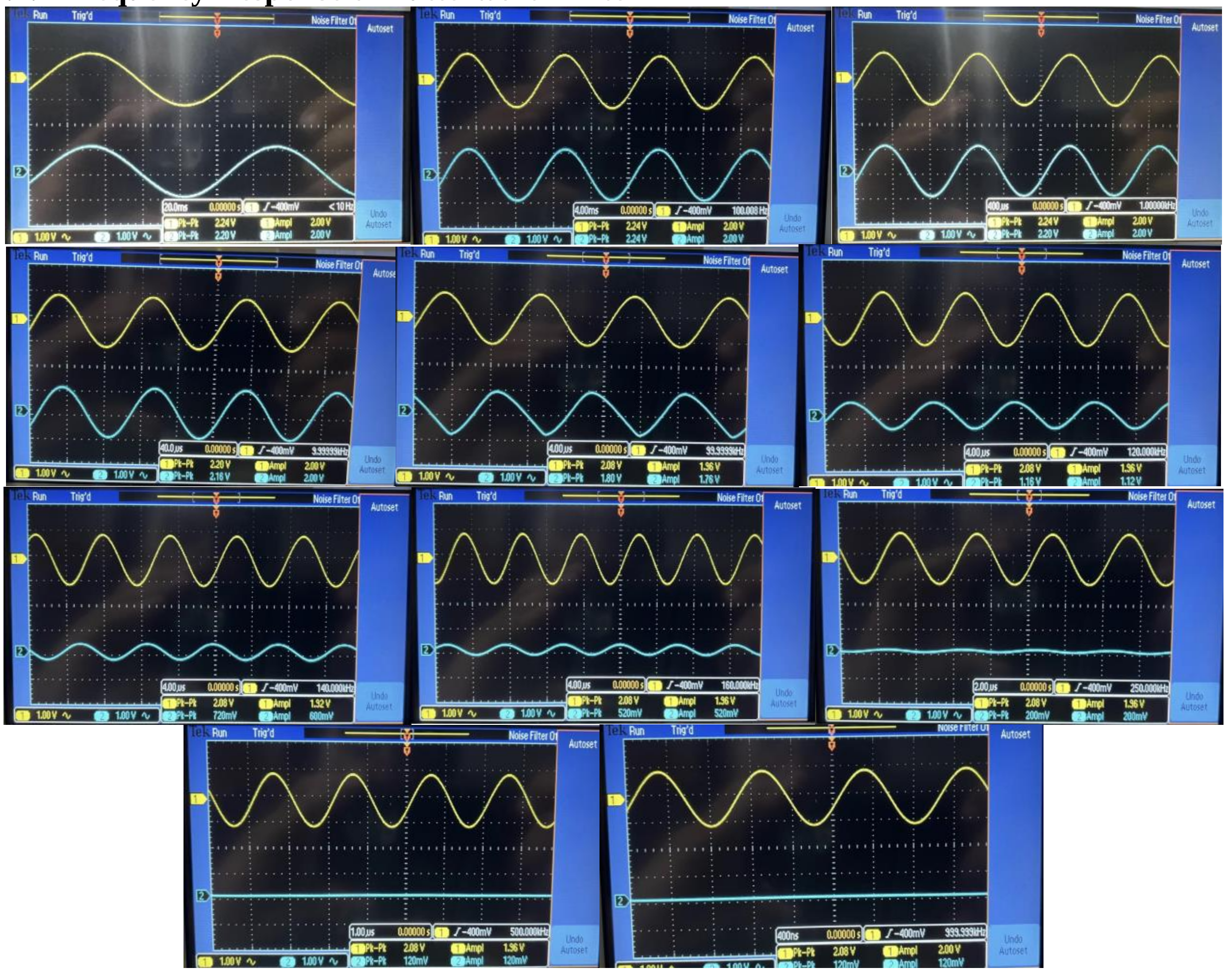


Figure 16 – Frequency Response from 10Hz to 1MHz

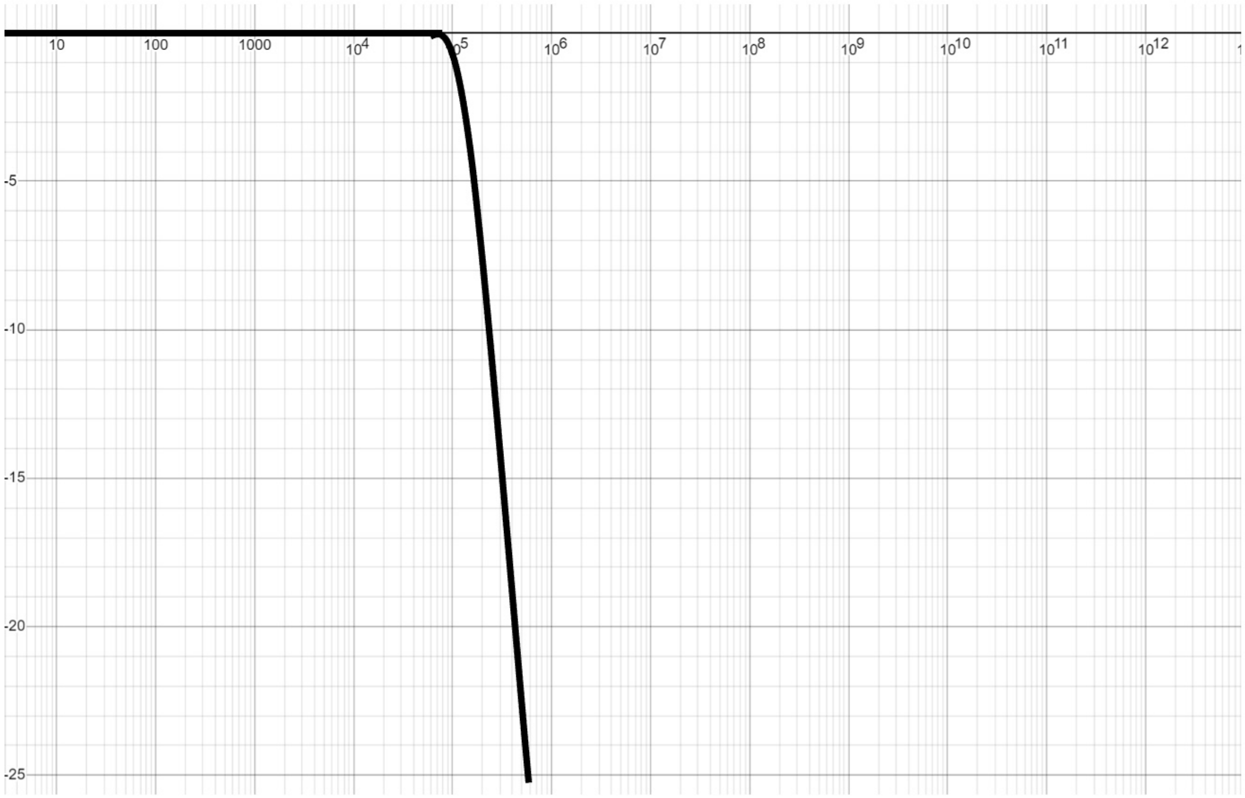


Figure 17 – Bode Plot of Gathered Frequencies

3. Encountered Problems

None

5. Conclusion

In conclusion, Butterworth Filters are a type of electronic filter widely used in audio & image processing, control systems, communications, and radar due to their desirable characteristics. They are known for their maximally flat frequency response in the passband region, with no ripples or phase distortion. There are multiple ways of creating Butterworth Filters, such as using the Sallen-Key topology using op-amps or using no op-amps using the Cauer Topology with only resistors and capacitors. However, for this project we decided to choose a nontraditional Butterworth Filter where the outputs of the Op Amps are cascaded into one another. From our design, we were able to get the same output result and frequency response as any typical Butterworth Filter. Butterworth filters can be implemented using software algorithms or hardware circuits, making them versatile and widely applicable in various domains.